Homework set 4

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1 Problem 1 - Cosmogenic nuclides

Cosmogenic nuclides profiles are used to date exposure surfaces. Here we will use a modified advection diffusion equation to solve for the production of nuclides, their decay and the moving free surface associated with either erosion or sedimentation. The equation we will be solving is

$$\frac{\partial C}{\partial t} = -\lambda C - S\frac{\partial C}{\partial z} + J_0 \exp(-\mu z), \tag{1}$$

where C is the nclide concentration, λ is the decay constant (=$4.6210e-07$ years^{-1} for ^{10}Be), S is either the sedimentation rate in cm/year (positive) or erosion rate (negative), J_0 is the surface production rate (=6) and μ is the characteristic penetration depth (=1/63).

(a) We will first focus on sedimentation ($S = +10^4$ cm/yr). Use a forward Euler time discretization with $\partial C(z)/\partial z = (C(z) - C(z-dz))/dz$. The domain you will solve the equation for is 2000 cm and you will discretize over 100 equally-spaced nodes ($dz = 20$ cm). Compute the evolution of the profile for 1e7 years with a timestep of 1e3 years. You need one boundary condition $C(z = 0, t) = 0)$ because sedimentation resets the profile at the surface. Use an initial condition $C(z, 0) = 0$. Plot the profile at the end of the calculation (time=1e7 years).

(b) Repeat the same exact calculation for erosion ($S = -10^4$ cm/yr), what happens? Why is it not working?

(c) Keep the same erosion rate but replace first the boundary condition to $C(z = 2000cm, t) = 0$ and then the spatial derivative to $\partial C(z)/\partial z = (C(z + dz) - C(z))/dz$. Plot the final profile, what do you observe?

Turn in a copy of your script and your results.

2 Problem 2 - High order ODE - Kepler's orbit

In this problem you will solve for Kepler's law of motion for a body in a gravity field. We will use Newton's law

$$\frac{d^2x}{dt^2} = \frac{F_x}{m} \tag{2}$$

$$\frac{d^2y}{dt^2} = \frac{F_y}{m}, \tag{3}$$

where F_x and F_y are the x and y force components and m is the mass of the orbiting object. The force vector (here we wil solve the trajectory in 2D - the ecliptic plane) is

$$\mathbf{F} = (F_x, F_y) = -(GmM/r^3)\mathbf{r}, \tag{4}$$

where $\mathbf{r}$ is the position vector (x, y) of the orbiting object (the reference being the Sun), r is its norm, G is the gravitational constant (=6.67e-11) and M the mass of the Sun. Use a set of coupled first order ODE's to solve for the trajectory of the orbiting object, first compute the velocity from the force and then the position from the velocity (integrating each time over time). The mass of the Sun is 1.988e30 kg, the mass of the Earth is 5.97e24 kg, the initial position for the orbiting object is $(1, 0)$AU, where AU=1.49e11 meters. Finally use an initial velocity $V(t = 0) = (0, 29.78e3)$.

(a) Write first a function file **Force.m** that requires 3 inputs: (i) the position of the orbiting object (vector with 2 components), (ii) M and (iii) m. This function turns back a 2 element vector F_x and F_y. Use a timestep $dt = 4000$ seconds. Run the model for 10'000 iterations. Plot (i) the trajectory of the orbiting planet (normalize the axes in AU), (ii) the position in direction x versus time for the object (normalize the time in years). Check that the orbit is done in 1 year (Earth's orbit).

(b) Repeat the same calculation but the initial position of the orbiting object is now $(0.4, 0)$AU. What is the orbital period now? Discuss the eccentricity of the orbit.

Turn in a copy of your script and your results (including plots).

3 Problem 3 - Poisson's equation

We will solve Gauss law in electrodynamics, which describe the electrical field associated with electrical charges.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \rho(x, y), \tag{5}$$

with $\rho(x, y)$ the density of charge that we will set to either ± 10 (in a charge) or 0 outside. The boundary conditions are zero potential $V = 0$ all around. Use a domain 40 by 40 nodes, with 3 charges with radius $r = 3$ nodes centered at positions (10,20), (30,30) and (20,30) with the first 2 are positive charges the last one negative.

Use the matrix method to solve this equation (be careful with indices), compute the solution for the potential V, the Electrical field $\mathbf{E} = (E_x, E_y) = (\partial V/\partial x, \partial V/\partial y)$ using the Matlab function **gradient**. Plot the potential and the vectorial electrical field (use the quiver function).